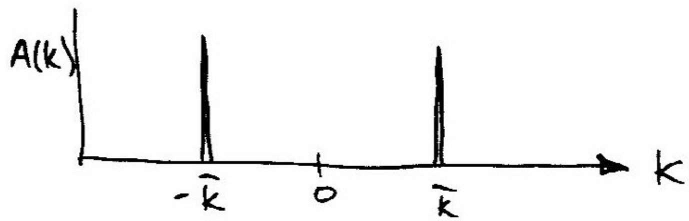
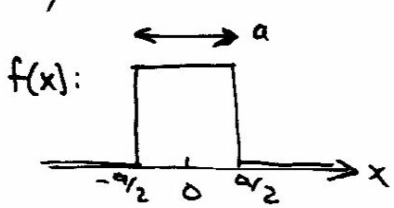
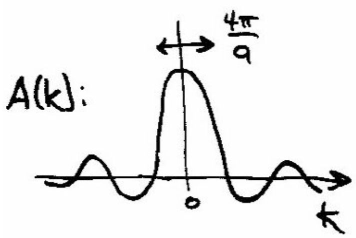
examples

a) $f(x) = \cos \bar{k}x$ or $\sin \bar{k}x$

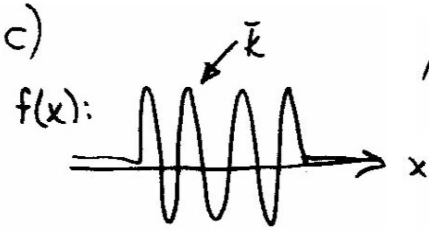


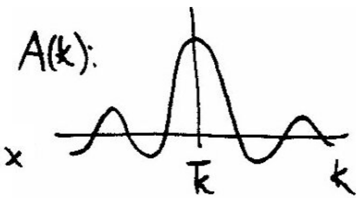
b)





c)





similarly at $k = -\bar{k}$

work out:

$$f(x) = \begin{cases} \cos \bar{k}x & -\frac{a}{2} < x < \frac{a}{2} \\ 0 & \text{elsewhere} \end{cases}$$

$$A(k) = \cdot \int_{-a/2}^{a/2} \cos \bar{k}x \, e^{-ikx} \, dx$$

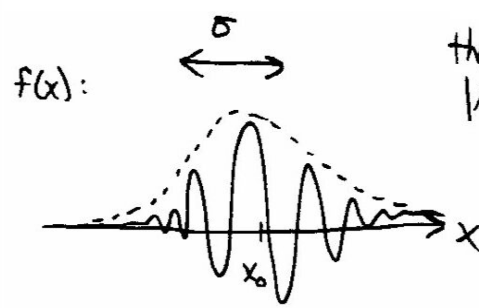
$$\cos \bar{k}x = \frac{1}{2}\left(e^{i\bar{k}x} + e^{-i\bar{k}x}\right)$$

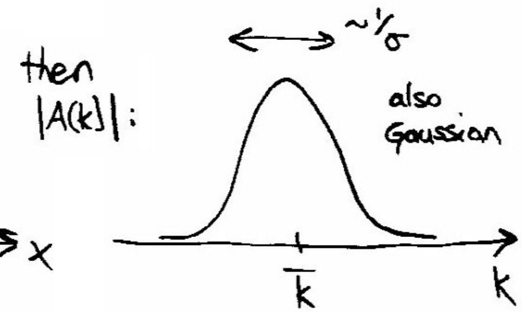
$$A(k) = \frac{1}{2} \int_{-a/2}^{a/2} \left(e^{-i(k-\bar{k})x} + e^{-i(k+\bar{k})x}\right) dx$$

$$= \frac{1}{2} \cdot \left[\frac{1}{-i(k-\bar{k})} e^{-i(k-\bar{k})x} \Big|_{-a/2}^{a/2} + \frac{1}{-i(k+\bar{k})} e^{-i(k+\bar{k})x} \Big|_{-a/2}^{a/2} \right]$$

$$= \frac{-}{\pi} \left[\frac{1}{k-\bar{k}} \sin \frac{(k-\bar{k})a}{2} + \frac{1}{k+\bar{k}} \sin \frac{(k+\bar{k})a}{2} \right]$$

d) Gaussian wavepacket

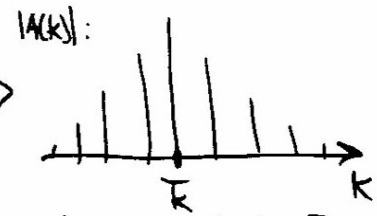




similarly at $k = -\bar{k}$

what if have periodic train of packets?

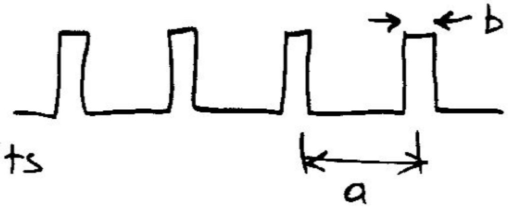
$\Rightarrow$



similarly at $k = -\bar{k}$

consider: <u>an array of finite slits:</u>

$s(x)$:



N slits

- can we derive this from previous result for single finite slit and N δ-fcn slits?

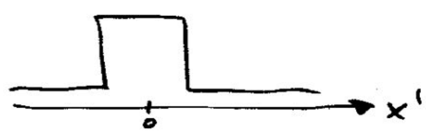
YES - use convolution (chap. 11)

Thm: if $g(x) = \int_{-\infty}^{\infty} dx' f(x') h(x-x')$

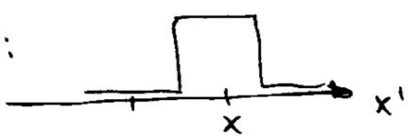
then $\tilde{g}(q) = \tilde{f}(q) \cdot \tilde{h}(q)$ Fourier transforms

example:

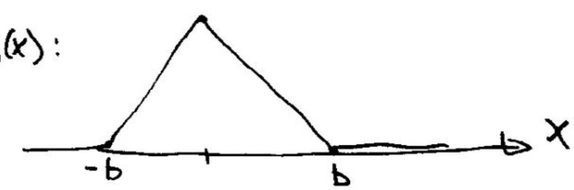
$f(x')$:



$h(x-x')$:

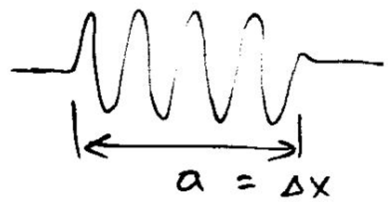


$g(x)$:

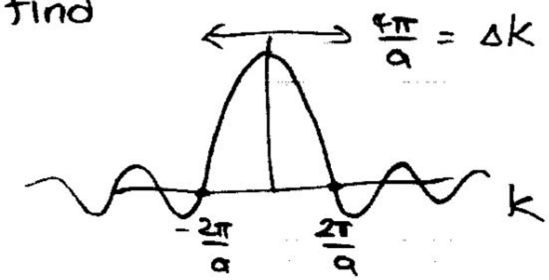


Bandwidth:

for



find



find $\Delta x \Delta k \approx 2\pi$ general result

bandwith.

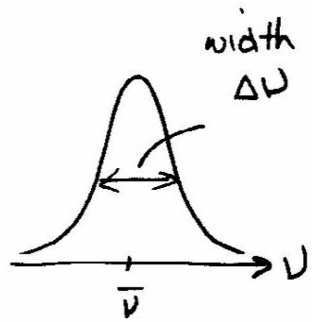
similarly $\Delta t \Delta \omega \approx 2\pi$,

$\Delta\omega = 2\pi \Delta\nu \Rightarrow \boxed{\Delta t \Delta \nu \approx 1}$

short $\Delta t \Rightarrow$ broad $\Delta\nu$

Coherence length

"monochromatic" source:



- width arises from
  - (i) natural decay time
  - (ii) Doppler shifts
  - (iii) collisions
  - ...

recall, for pulse width Δt, full width at half-max of F.T. is $\Delta\omega = \frac{2\pi}{\Delta t}$,

or $\Delta\nu \, \Delta t \approx 1$.

define coherence time $\Delta t_c = \frac{1}{\Delta\nu}$

coherence length $\Delta l_c = c\,\Delta t_c$

Δt_c - time between random phase shifts

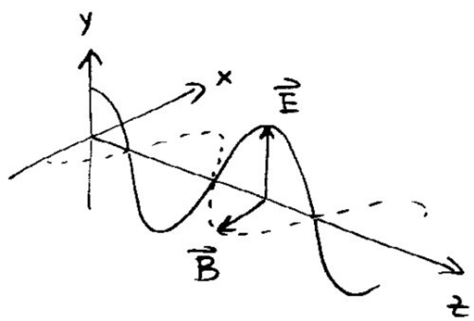
$\Delta l_c \sim .1$ m discharge lamp

100 m He-Ne laser.

Polarization

eg/ linearly polarized $\vec{E} = \hat{j} E_{oy} e^{i(kz-\omega t)}$

$$= \vec{E}_o e^{i(kz-\omega t)}$$



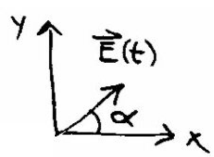
$$\vec{E}_o = \begin{bmatrix} 0 \\ E_{oy} \end{bmatrix} = E_{oy} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Jones vector

similarly, for

$$\vec{E}_o = E_{ox} \hat{\imath} \quad , \quad \vec{E}_o = \begin{bmatrix} E_{ox} \\ 0 \end{bmatrix} = E_{ox} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

or in general, for



$$\vec{E}_o = E_o \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}$$

$$\alpha = \tan^{-1} \frac{E_{oy}}{E_{ox}} \quad , \quad E_o = \sqrt{E_{ox}^2 + E_{oy}^2}$$

also, can have circular polarization $\nearrow$

$$\vec{E}_o = \frac{E_o}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix} \quad , \quad \vec{E} = \frac{E_o}{\sqrt{2}} \left(\hat{\imath} e^{i(kz-\omega t)} + \hat{j} e^{i(kz-\omega t+\frac{\pi}{2})} \right)$$

$$i = e^{i\pi/2}$$

or

$$Re(\vec{E}) = \frac{E_o}{\sqrt{2}} \Big(\hat{\imath} \cos(kz-\omega t) + \hat{j} \underbrace{\cos\left(kz-\omega t+\frac{\pi}{2}\right)}_{\cos\left(kz-\left(\omega t-\frac{\pi}{2}\right)\right)} \Big)$$

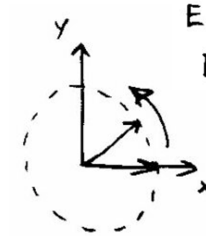
$$= -\sin(kz-\omega t)$$

consider at $z=0$:

$$Re(E_x) = \frac{E_o}{\sqrt{2}} \cos(\omega t)$$

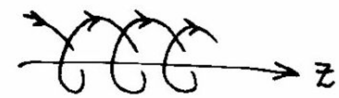
$$Re(E_y) = \frac{E_o}{\sqrt{2}} \sin(\omega t)$$

traces out a circle going ccw with time



E_y lags E_x

at fixed time:



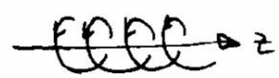
$\vec{E}$ traces out LH spiral

$\Rightarrow$ left circularly polarized

and for

$$\vec{E} = E_o \begin{bmatrix} 1 \\ -i \end{bmatrix}$$

have right circular polarization



RH spiral or

CW with time

consider sum of RCP and LCP waves,

$$\vec{E} = \frac{E_0}{\sqrt{2}} e^{i(kz-\omega t)} \left[\hat{\imath} + \hat{\jmath} e^{i\pi/2} + \hat{\imath} - \hat{\jmath} e^{i\pi/2} \right]$$

$$= \sqrt{2} E_0 \, e^{i(kz-\omega t)} \hat{\imath}$$, linearly polarized

with Jones vectors,

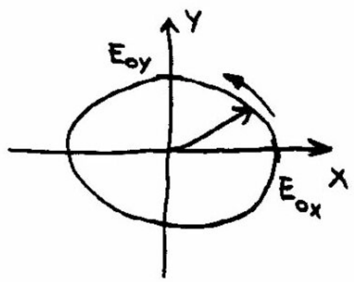
$$\frac{E_0}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix} + \frac{E_0}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix} = \sqrt{2} E_0 \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \checkmark$$

what if mag. of E_{ox} and E_{oy} unequal?

$\Rightarrow$ elliptical polarization

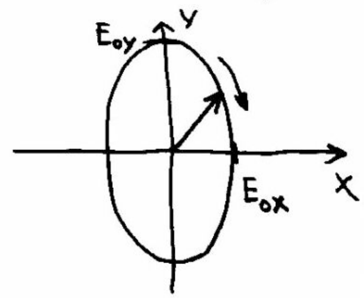
$$\frac{E_0}{\sqrt{E_{ox}^2 + E_{oy}^2}} \begin{bmatrix} E_{ox} \\ iE_{oy} \end{bmatrix} \quad \text{ccw L.E.P.}$$

eg/ for $E_{ox} > E_{oy}$



$$\frac{E_0}{\sqrt{E_{ox}^2 + E_{oy}^2}} \begin{bmatrix} E_{ox} \\ -iE_{oy} \end{bmatrix} \quad \text{cw R.E.P.}$$

eg/ for $E_{ox} < E_{oy}$

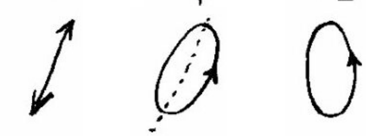


• in prior example still have $\pi/2$ phase shift between E_x and E_y; in general may have arbitrary phase shift:

$$\frac{1}{\sqrt{E_{ox}^2 + E_{oy}^2}} \begin{bmatrix} E_{ox} \\ E_{oy} e^{\pm i\varepsilon} \end{bmatrix} \qquad \begin{matrix} +, \text{ ccw (LEP)} \\ -, \text{ cw (REP)} \end{matrix}$$

consider + sign,
$E_{oy} = 2E_{ox}$

ε: 0 $\quad \pi/4 \quad \pi/2$

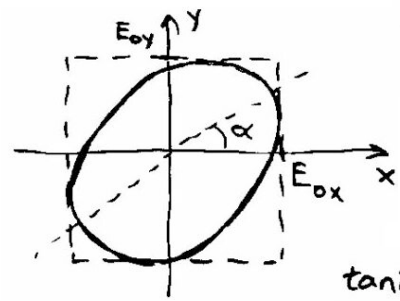


for $\varepsilon = \pi/4$

$$\vec{E}_0 = \frac{E_0}{\sqrt{5}} \begin{bmatrix} 1 \\ 2e^{i\pi/4} \end{bmatrix}, \quad \vec{E} = \frac{E_0}{\sqrt{5}} \left(\hat{\imath} e^{i(kz-\omega t)} + 2\hat{\jmath} e^{i(kz-\omega t+\pi/4)} \right)$$

$$\mathrm{Re}(\vec{E})\Big|_{z=0} = \frac{E_0}{\sqrt{5}} \left(\hat{\imath} \cos(\omega t) + 2\hat{\jmath} \cos(\omega t - \pi/4) \right)$$

in general,



$$\tan 2\alpha = \frac{2E_{ox}E_{oy}\cos\varepsilon}{E_{ox}^2 - E_{oy}^2}$$

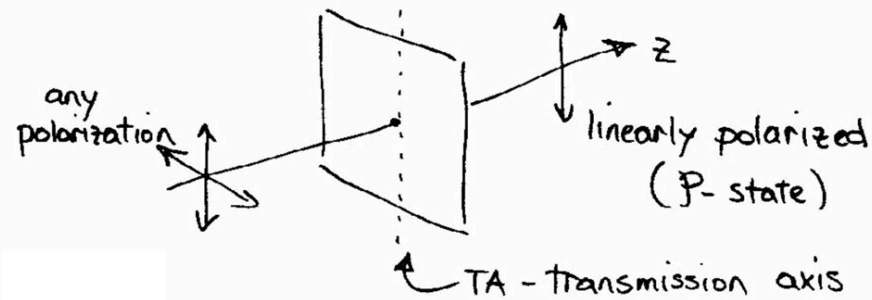
above example:

$$\tan 2\alpha = \frac{4\cos\pi/4}{1-4} \Rightarrow \alpha = 68^\circ$$

unpolarized light - direction of $\vec{E}$ changes irregularly (randomly), with coherence time Δt_c.

Devices for changing polarization

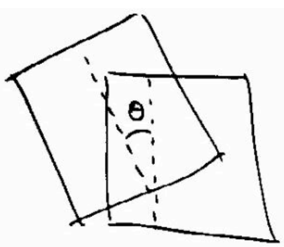
linear polarizer



examples:

- dichroic material - absorbs $\vec{E}$ component along particular direction (eg. linear conducting polymers).
- for microwaves, an array of wires.

crossed polarizers:



$I = I_o \cos^2\theta$

Malus's law

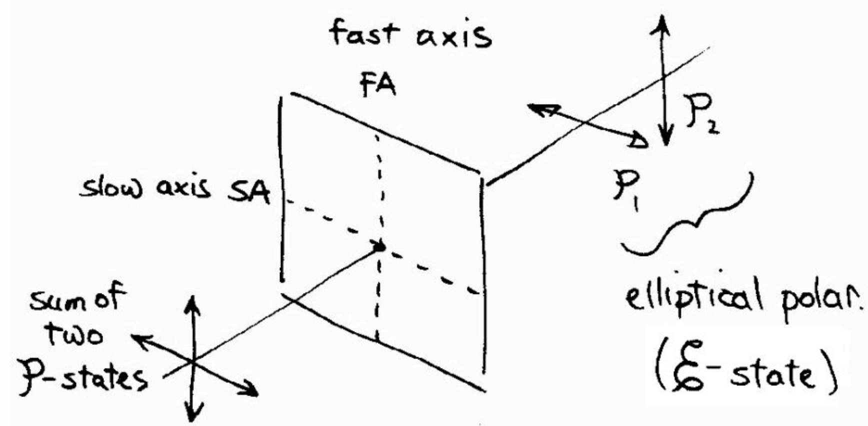
Jones matrix:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$ matrix is $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ for TA along $\hat{j}$.

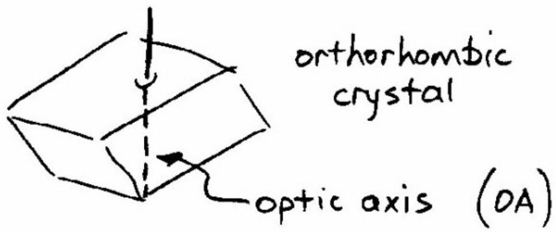
or, $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ for TA along $\hat{\imath}$, etc.

phase retarder

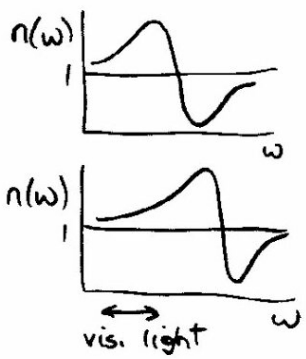


eg/ birefringent crystals - different indices of refraction depending on polarization.

calcite:



different absorption bands for polarization parallel or perpendicular to OA.



eg $\lambda = 589$ nm (Na)

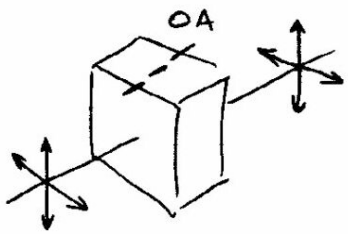
$n_{\parallel} = 1.486$

$n_{\perp} = 1.658$

$\Rightarrow v_{\parallel} > v_{\perp}$

say, have:

two P-states both $\perp$ to OA

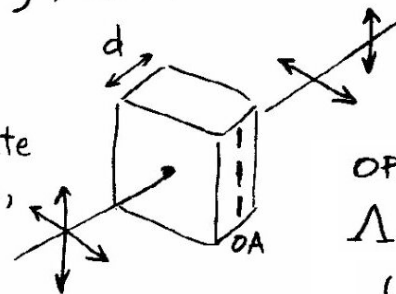


no effect

both P-states have polar. $\perp$ to OA

alternatively, with

one P-state // to OA, one $\perp$.

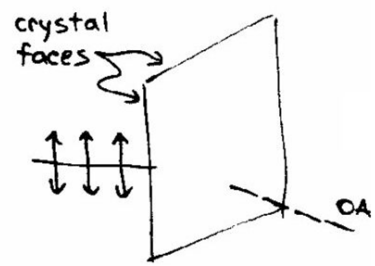


OPD

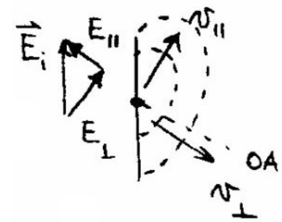
$\Lambda = d(n_{\perp} - n_{\parallel})$

$\delta = k\Lambda$

for polarization neither // or $\perp$ to OA, things are more complicated:

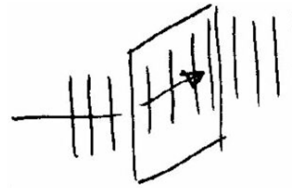


use Huygen's principle:



secondary wavefronts are ellipsoidal!

wavefront propagation:



($\vec{k}$ not $\perp$ to wavefront!)

in general: